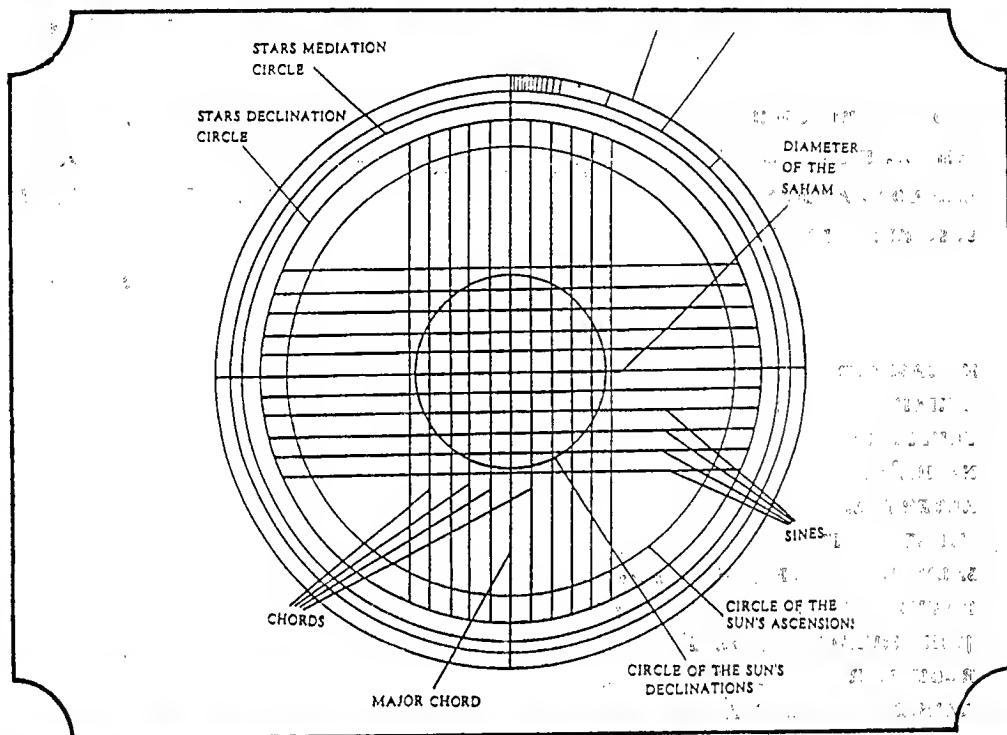


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TRANSFORMATION OF COORDINATES IN IBN BĀṢO'S *AL-RISĀLA FĪ'L-ṢAFĪḤA AL-MUJAYYABA DHĀT* *AL - AWTĀR*

Emilia Calvo**

1 . Introduction :

Manuscript 5550 in the National Library (Bibliothèque National) in Tunis contains (in pages 50 r - 81 v) a treatise on the use of what is called *al - ṣafiḥa mujayyaba dhāt al - awtār* (plate of sines provided with chords) . The author of the treatise is mentioned on the first page . His complete name is Abū ʿAlī Ḥusayn ibn Abī Jaʿfar (Aḥmad) ibn Yūsuf ibn Bāṣo al- Aslamī and he is described as *imām al-muʾadhdhinin* (chief of the muezzins) *qudwat al - muʿaddilīn* (an example for astronomers), and *amīn awqāt al - salawāt* (in charge of the times of prayer) in Granada¹ .

On page 81 the name of the copyist is also given Muḥammad b. Muḥammad , called Abū - l Ṣagīr , b. al-Ḥāy b. ʿA bd Allah Muḥammad b.al - Ḥāy b.al - Faḍl Qāsim , and the date of copy : 1305 H. which corresponds to 1887 A.D. Therefore it is a rather late copy but this is , to the best of my knowledge , the only one preserved today .

There is no date of composition but this Ibn Bāṣo was also the author of a treatise on the use of a device that he called *al - ṣafiḥa al - jāmiʿa li - jam ʿi ʿal - ʿurūd* (universal plate for all latitudes) in 160 chapters . This treatise , completed in 673 H./ 1274 A.D., is preserved in three extant manuscripts now in El Escorial (MS ar . 961) , The National Library of Tunis (MS 9215) and the Royal Library of Rabat (MS 4288). The author of this second treatise is

* - Paper given at The Sixth International Symposium for The History of Arabic Science.
RAS AL - KHAIMAH , 16 - 19 December , 1996 .

** - University of Barcelona .

1 - On this autour cf. SARTON, vol , III P. 696; SUTER, n°381 b, p.157; BROCKELMANN. S I, p. 869 ; SANCHEZ PEREZ, p.79; RENAUD 1932, n°381 b, p.172; RENAUD 1937, pp.1-12; MILLAS VALLICROSA 1933, pp. XXXVII-XXXVIII; MILLAS VALLICROSA 1943 - 1950 , pp. 448 - 449 ; SAMSO 1973 , pp. 176 - 182.

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referred to in the first page and his complete name coincides in both cases .This second treatise was edited translated and studied several years ago . It shows that the author was aware of the *mīqāt* procedures already in use among the muwaqqits of his time specially in Egypt and Syria ².

Ibn Bāšo's universal plate was widely diffused as we learn from the abridgements of his treatise still preserved ³and , on the other hand , from the knowledge of this kind of projection among several astrolabe - makers who included it in their works . This , at least , can be deduced from the number of the extant plates , mainly in al - Andalus and North Africa , but also in the Muslim East .

This Ibn Bāšo seems to be one of the two astronomers to whom Ibn al - Khaṭīb devotes some lines in his biographical work entitled *Al - Ihāṭa fī akhbār Garnāṭa'* . In those lines Ibn al - Khaṭīb describes our author as *shaykh al - jamā'c* and says that he is the author of inventions and treatises but these two treatises are the only preserved as far as I know .

2 - Description of the instrument :

The treatise on the use of the plate of sines contains 59 chapters . The first chapter is devoted to the description of the lines traced on the plate .This description is rather vague in some passages . In most cases the author limits himself to giving the name of the elements , without any indication of their appearance . Unfortunately there are no drawings on the text to clarify the nature of some of the lines engraved on the instrument . For instance , Ibn Bāšo states that the back of the instrument is similar to the back of the astrolabe, but gives no further explanation .

Ibn Bāšo mentions the following elements on the face of the instrument (fig . 1) ⁴ .

- first, the outer circle (or external circumference *dā'irat al-muḥīṭ*), which is divided into 360 degrees .According to the author , this circumference can correspond to the circle of the equator , the horizon circle , the meri-

2 - Cf . Calvo , 1993 .

3 - Cf . CALVO 1991 .

4 - Cf . IBN AL - KHATĪB, vol . I , p.204.

5 - Cf . fol . 51 v - 52 v

dian circle, the altitude circle and the ecliptic, depending on the operation to be performed .

- second , an inner circle , which is divided into four quadrants . In each quadrant that Ibn Bāṣo describes , every one of its ninety degrees are marked on the circle. The first quadrant is from the beginning of Capricorn to the end of Pisces. The second part goes from the beginning of Aries to the end of Gemini , and is also divided in 90 degrees . The third is not described in the text . It should go from the beginning of Cancer to the end of Virgo , and the fourth goes from the beginning of Libra to the end of Sagittarius , and is also divided in 90 degrees .

Ibn Bāṣo states that the degree markings can be used for the ecliptic and the outer circle as well .

- then , the stars' mediation circle inside the ecliptic and , the stars' declination inside the mediation circle .

The text states that the stars are depicted between the mediation and the declination circles , and their names are written there as well . The names of the zodiacal signs are also found between these circles and the ecliptic .

the following element in the plate is the diameter of the *sahām* which is taken from the beginning of Capricorn to the beginning of Cancer .

the chords are the straight lines which cut the diameter of the *sahām* . Among them is the major chord which is the diameter taken from the beginning of Aries to the beginning of Libra .

the sines are the straight lines which cut the chords .

The graduation of the *sahām* is written on its diameter . It starts at the beginning of Capricorn and finishes at the beginning of Cancer .

The graduation of the sines is written on the major chord . It starts at the centre and goes to the beginning of Aries from 0 to 90 and to the beginning of Libra also from 0 to 90 .

the text also mentions a *muri* (index) moving along a *khayṭ* (thread), which describes circles on the plate .

Finally the circle of the sun's declinations and the circle of the sun's ascensions are mentioned. The author says that the former is smaller, but gives no

other explanation of its situation on the plate . However , from the analysis of the instructions on the use of the plate which are given in the treatise for transforming coordinates , it is possible to determine the way in which these two circles are drawn . This analysis is one of the main objectives of this paper . From this description it is possible to reconstruct the aspect of the face of the instrument as seen in fig . 2.

3 - Contents of the treatise :

The use of this plate is described in chapters 2 to 59 . Ibn Bāṣo describes the following topics:

- 2 - 3 : Determination of sines cosines and chords.
- 4 - 12: Determination of the solar degree , the latitude , the meridian altitude and the declination .
- 13 - 16: Determination of shadows .
- 17 : Determination of the diurnal times of prayer: *al - ẓawāl* , *al - zuhr* and *al-ʿaṣr*.
- 18 - 20: Determination of the diurnal and nocturnal arcs and of the seasonal and equal hours .
- 21 - 28: Determinations related to the time elapsed since sunrise .
- 29 - 32: Calculations related to stars .
- 33 : Determination of morning and evening twilight .
- 34 - 35: Determination of the altitude of a star from the hour and vice versa .
- 36 - 42: Determination of the azimuths .
- 43 : Determination of longitudes .
- 44 - 53: Determination related to stars .
- 54 - 58: Determination of right and oblique ascensions .
- 59 : Determination of whether a planet is direct or retrograde .

As we can see , the first chapters are devoted to trigonometric calculations by means of plane and sphetical trigonometry . The next chapters are devoted to questions related to religious purposes which is probably his principal interest . As I have said he is described in the introduction of the treatise as *imām al - mu'adhdhinīn* and *amīn awqāt al - ṣalwāt* in Granada . He describes

how to calculate the times of the diurnal prayers (and also the nocturnal ones by determining the time of morning and evening twilight) .

4 - Transformation of coordinates in Ibn Bāso's plate of sines :

Ibn Bāso deals with this point in chapters 5 to 12 in which he describes how to obtain the degree of the equator from the degree of the ecliptic , that is to say , the right ascension from the longitude (in chapter 5) , and also the reverse procedure (in chapter 6) . He explains how to calculate the degree of the ecliptic from the solar declination and the reverse process (in chapters 7 to 9) ; how to obtain the latitude of a place from the meridian altitude and the declination of the sun on that day and the reverse process: the meridian altitude from the declination and the latitude of the place (in chapters 10 to 12) .

Ibn Bāso gives geometrical explanations but there are underlying trigonometrical procedures as we can see in the following examples :

In chapter 5 the right ascension is calculated from the degree of the , ecliptic . Ibn Bāso's instructions are (fig.3) to measure the solar longitude , λ , from point G which is the beginning of Aries . He obtains point D . The thread of the instrument will intersect the circle of ascensions , $A'B'G'K$, at a point F which determines a chord of the plate , EH which in turn , will intersect the sine determined by the extreme of the thread DI . This intersection , point H , gives the position of the thread OH which will determine point M on the outer circle , Arc GM , which equals angle GOM , will be the value of the right ascension , α , corresponding to the solar longitude λ given ° .

The demonstration is as follows :

In triangle OEF $\sin \lambda = OE / OF$

and , since $OF = R \cos \epsilon$, then

$$OE = R \sin \lambda \cos \epsilon$$

In triangle ODI

$$\cos \lambda = OI / R \quad \text{then} \quad OI = R \cos \lambda$$

In triangle OIH

$$\tan \alpha = IH / OI \quad \text{and , since } IH = OE \text{ , by substitution}$$

$$\tan \alpha = \tan \lambda \cos \epsilon \quad (= R \sin \lambda \cos \epsilon / R \cos \lambda)$$

which is the underlying formula .

Chapter 6 describes the reverse procedure , that is to say , determining the degree of the ecliptic λ , from the degree of the equator (fig . 4) . In this case Ibn Bāso makes the thread coincide with a degree of the equator GM and determines the sine of the plate , line IH , which crosses the intersection of the thread and the circle of ascensions L . This sine line also intersects the chord corresponding to point M , namely ME , at point N . Then , the thread is placed on point N determining point P on the external circle which provides the value of the corresponding degree of the ecliptic ⁷ .

Once again the underlying formula is $\tan \alpha = \tan \lambda \cos \epsilon$ if we consider the ascension circle to have a radius $OK = R \cos \epsilon$.

In chapters 7 and 8 the author determines the declination from the longitude (fig . 5) . The value λ determines point L . The thread is placed over this point and intersects the circle of declinations at point M . This point determines a chord ND , the intersection of which with the outer circle D gives the value of the solar declination GD .

Another procedure for obtaining the chord DN is to determine first chord LS , which passes through point L . Then , the index of the thread is placed over the intersection of the chord with the diameter of the plate , S . And then , the thread is placed according to the value of the obliquity of the ecliptic GE . The index intersects the chord DN , thus giving the value of the solar declination ⁸.

In this case the underlying formula is

$$\sin \lambda = \sin \delta / \sin \epsilon$$

In chapter 9 , which is the reverse of chapters 7 and 8 , Ibn Bāso describes how to obtain the degree of the ecliptic from the solar declination. In this case he places the alidade according to the value of the declination , $\delta = GD$. The point of intersection of the chord DMN with the circle of declination determines a point , M , through which the alidade must pass to determine on the limb the arc GL corresponding to the value of the solar longitude ⁹.

7 - Cf . fols , 54 v - 55 r .

8 - Cf . fols , 55v - 56 r

9 - Cf , fol , 56 r .

Here the circle of declination is introduced . Its radius equals $r = R \sin \varepsilon$.

In fig . 5 we have that :

in triangle ODN

$$ON = R \sin \delta$$

in triangle OMN

$$\sin \lambda = R \sin \delta / r = R \sin \delta / R \sin \varepsilon$$

therefore we have that

$$\sin \lambda = \sin \delta / \sin \varepsilon$$

In chapter 10 the latitude is determined from the meridian altitude . Here the author applies to the plate the formula $h = 90 - \varphi + \delta$.

In this chapter we find a coincidence with something found in the treatise on the universal plate . The author takes into account the possibility of the sun beign situated to the north of the pole . This happens when the latitude is smaller than the declination ($\varphi < \delta$) . In this case h would be greater than 90 degrees , which is impossible . The solution is to obtain $h' = 180 - h = 90 + \varphi - \delta$. In this case he operates in the opposite quadrant of the instrument (fig . 6)¹⁰.

In chapter 11 the meridian altitude is determined from the longitude or the declination , In chapter 12 the longitude and the declination are determined from the meridian altitude .

5 - Abridged reference :

In short , this plate makes it possible among other things , to transform coordinates and to solve problems of spherical astronomy avoiding all the trigonometric calculations involved which are implicit in the use of the instrument .

From the indications given by the author it is possible to determine how the ascension circle and the declination one are drawn. The ascension circle has a radius $r_1 = R \cos \varepsilon$ and the declination circle has a radius $r_2 = R \sin \varepsilon$.

A brief schema of the topics dealt with in the corresponding chapters is given below . In this analysis the trigonometrical formulae involved are ;

Chap. 5	:	$\alpha (\lambda)$	$\tan \alpha = \tan \lambda \cos \varepsilon$
Chap 6	:	$\lambda (\alpha)$	$\tan \lambda = \tan \alpha / \cos \varepsilon$
Chap 7- 8	:	$\delta (\lambda)$	$\sin \delta = \sin \lambda \sin \varepsilon$
Chap 9	:	$\lambda (\delta)$	$\sin \lambda = \sin \delta / \sin \varepsilon$

10- Cf . fol . 56r - 56v .

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Chap 9	:	$\lambda (\delta)$	$\sin \lambda = \sin \delta / \sin \epsilon$

10- Cf . fol . 56r - 56v .

Chap 10	:	$\varphi (\delta , h)$	$\varphi = 90 + \delta - h$
Chap 11	:	$h (\lambda (\delta) , \varphi)$	$h = 90 - \varphi + \delta$
Chap 12	:	$\lambda , \delta (h , \varphi)$	$\delta = h + \varphi - 90$

α : right ascension

λ : ecliptic longitude

ε : obliquity of the ecliptic (maximum declination)

φ : latitude of the place

δ : declination

h : meridian altitude

6 - *Precedents of Ibn Bäs's plate of sines* :

As for the precedents of Ibn Bäs's plate of sines , what can be said is that from the description of this plate it is evident that it is related to the sine quadrants constructed in the Islamic East from the ninth century onwards . The first known description of this instrument was by al - Khwārizmī , following his two treatises on the astrolabe ¹¹.

In the beginning , the quadrant was an independent instrument , but from the tenth century onwards it was found on the back of the standard astrolabe .

Ibn al - Zarqālluh was the first astronomer to introduce the sine quadrant on the back of an astronomical instrument in al - Andalus , with the instrument called *al - ṣafiḥa al - zaqāliyya* . Ibn Bäs knew Ibn al Zarqālluh's work as is evident from the treatise on the use of his universal plate (called *al - ṣafiḥa al - jāmi'a li - jamī' al - 'urūd*) . It is possible that some elements in the treatise on the *al - ṣafiḥa al - mujayyaba* come from Ibn al Zarqālluh . But there is a difference since Ibn Bäs is describing a complete plate , not only a quadrant .

This plate is probably related to the instrument called the *dastūr* circle , an instrument described by several astronomers in the Islamic East . Among them are Abū Sa'īd ['Abd al - Raḥmān b . Abī ḤaFṣ 'Omar b . Muḥammad] al - Abharī from the thirteenth century (d . A. D. 1274)¹². According to Schmalzl , he may have been the inventor of this instrument which is described in his work entitled *Lawāmi' al - wasā' il fi - maṭāli' al - rasā'il* preserved in MS. 965 (according to Renaud)¹³ of EI Escorial and in Gotha 1414 . The third maqāla

11- Cf . KING , 1983 , PP. 29 - 30

12- Cf . SUTER n . 369 , SCHMALZL , PP. 62 - 63 .

13- Cf . RENAUD 1941 , p . 109

is divided into 40 chapters devoted to the *dastūr* .

This instrument was also described by Abū - l - Ḥasan ^cAlī al - Marrākuṣī , an astronomer from the thirteenth century who worked in Egypt¹⁴, He describes it in his treatise entitled *jāmi^cal - mabādi' wa - l - gāyāt fī ^cilm al - miqāt* .

The *dastūr* is also described by a muwaqqit in al - Azhar , Jamal al - dīn al - Maridīnī (m. 1406)¹⁵ , in his treatise *Al - durr al - manthūr fī l - ^camal bi - l - rub^cal - dastūr* which is preserved in several copies now in Berlin 5840 , Madrid (Esc . 931) , Oxford , Paris Cairo .¹⁶ 'This treatise contains 60 chapters'¹⁷.

At the end of the XVth century an instrument called *sexagenarium* was introduced in al - Andalus .¹⁸ This instrument had two faces , one planetary and one trigonometric , Of arabic origin , it was used by the *muwaqqits* in Egypt and reached the Iberian peninsula in 1450 via a *faqīh* from Paterna who was in the city of Valencia at that time . This instrument was later introduced in Europe . There are two translations of the treatise describing its use , one of which was printed . This instrument was described by Poulle several years ago and presents many points of contact with Ibn Bāso's plate of sines¹⁹.

7 - Concluding remarks

To conclude this study I would like to add two observations , one related to the instrument itself and the other to its place in the history of the trigonometrical instruments :

First , Ibn Bāso does not specify in the treatise the value of the obliquity of the ecliptic . But this value is not needed , since there are two circles which are traced according to it : the circle of declinations and the circle of ascensions , with which he operates when he needs to introduce this value . In the description of the instrument these two circles are introduced in a rather vague way , but a

14- Cf . SUTER n . 363.

15- Cf . SUTER n.421.

16- Sibṭ al - Maridīnī's grandfather . Sibṭ is also the author of a treatise on the use of the sine quadrant which is contained in the same Ms. 931 Escorial .

17- Cf . SEDILLOT p.88 n . 2.

18- On this instrument cf . THORNDIKE, pp. 130 - 133 .

19- On the characteristics of the *sexagenarium* cf . POULLE 1966 , pp. 129 - 161 ; POULLE 1980 , I pp. 417 - 444 and SAMSO 1992 , pp. 216 - 217 .

study of these chapters reveals how they are traced on the plate : the circle of declinations has a radius of value $(R \sin \epsilon)$ and the circle of ascensions has a radius of value $(R \cos \epsilon)$ R being the radius of the plate which , as I have said , is divided into 60 parts , and ϵ the value of the obliquity of the ecliptic adopted by the author . But this seems to be a rather technical question and the treatise tends to avoid them in order to make it useful for non specialists .

Secondly , This instrument can be inscribed in the stream of influences East - West - East found between al - Andalus , North Africa and the Islamic East from the eleventh century onwards .

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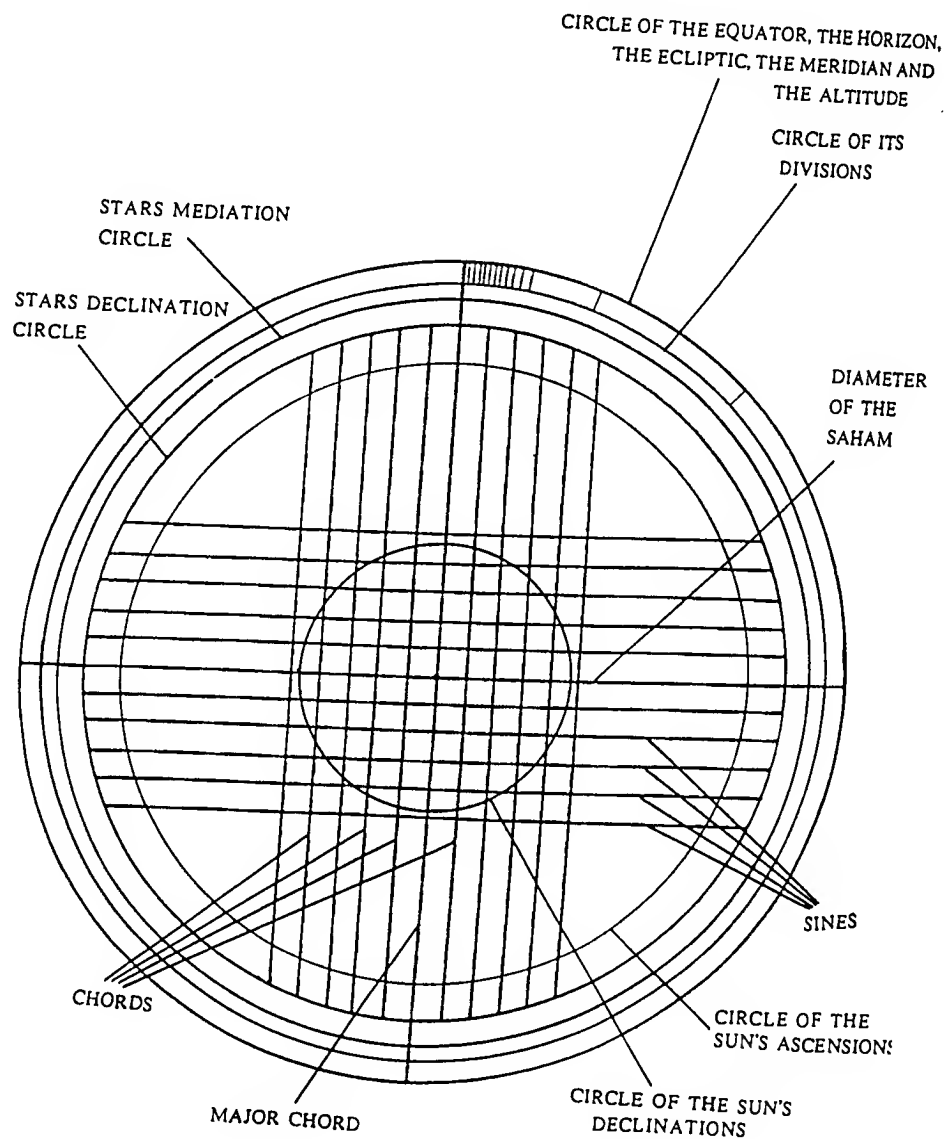


Fig. 1
Ibn Bāṣo's *al-ṣafiha al-mujayyaba dhāt al-awṭār*
(Elements)

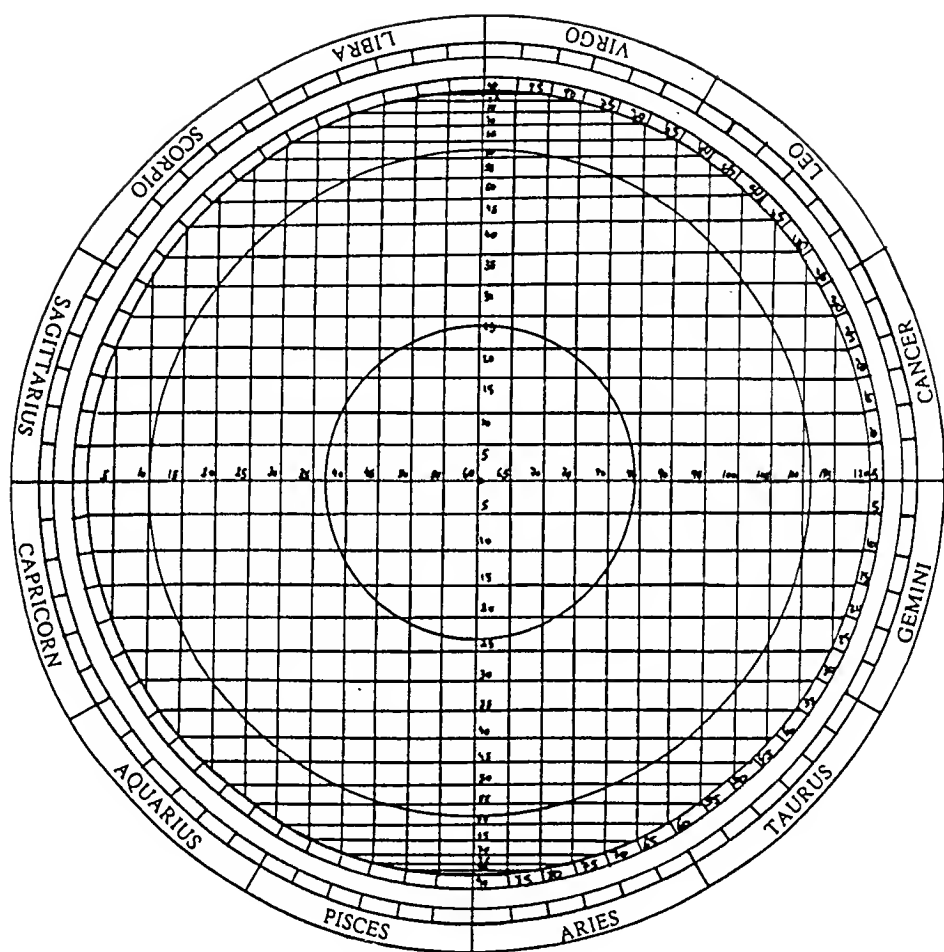
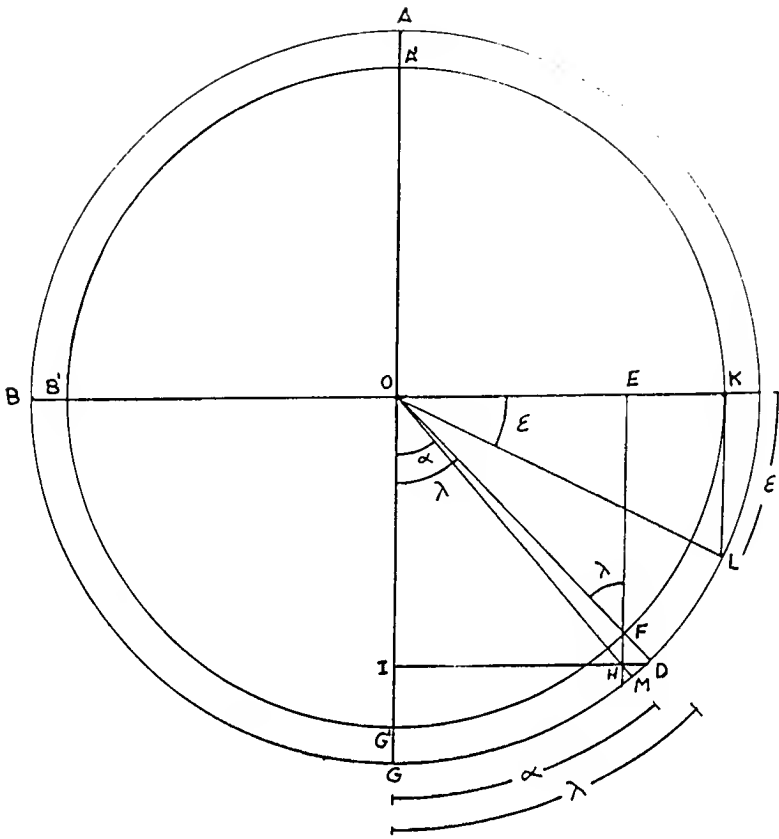


Fig. 2

Ibn Bāṣo's *al-ṣafīḥa al-mujayyaba dhāt al-awṭār*
(Reconstruction)



In OLK

$$OK = R \cos \epsilon$$

In OEF

=>

$$\sin \lambda = OE/OF \quad \text{and} \quad OF = R \cos \epsilon$$

$$OE = R \sin \lambda \cos \epsilon$$

In ODI

$$\cos \lambda = OI/R \quad \Rightarrow \quad OI = R \cos \lambda$$

In OIH

$$\tan \alpha = IH/OI \quad \text{and} \quad IH = OE = R \sin \lambda \cos \epsilon$$

=>

$$\tan \alpha = \tan \lambda \cos \epsilon$$

Fig. 3

